دانشگاه آزاد اسلامی واحد تبریز

نام درس: داده کاوی

بخش: قوانین انجمنی

نام استاد: دکتر مسعود کارگر

Roadmap

- Frequent Itemset Mining Problem
- Closed itemset, Maximal itemset
- Apriori Algorithm
- FP-Growth: itemset mining without candidate generation
- Association Rule Mining

Case 1: D.E.Shaw & Co.

- **D. E. Shaw & Co.** is a New York-based investment and technology development firm. By Columbia Uni. CS faculty.
  - manages approximately US $35 billion in aggregate capital
  - known for its quantitative investment strategies, particularly statistical arbitrage
  - **arbitrage** is the practice of taking advantage of a price differential between two or more markets
  - statistical arbitrage is a heavily quantitative and computational approach to equity trading. It involves data mining and statistical methods, as well as automated trading systems

StatArb, the trading strategy

- StatArb evolved out of the simpler pairs trade strategy, in which stocks are put into pairs by fundamental or market-based similarities.
- When one stock in a pair outperforms the other, the poorer performing stock is bought long with the expectation that it will climb towards its outperforming partner, the other is sold short.

Example:
PetroChina
SHI
CEO

http://en.wikipedia.org/wiki/Statistical_arbitrage

StatArb, the trading strategy

- StatArb considers not pairs of stocks but a portfolio of a hundred or more stocks (some long, some short) that are carefully matched by sector and region to eliminate exposure to beta and other risk factors
- Q: How can u find those matched/associated stocks?
- A: Frequent Itemset Mining -☺

Transaction records:
S1↑S2↓S3↓S4 ↑
S1↑S2↓S3↑ S4↑
S1↓S2↑S3↓S4 ↓
S1↑S2↓S3↑S4 ↑

(S1, S2) → S1 ↓ S2 ↑ → Buy S1

Case 2: The Market Basket Problem

Market-Basket transactions

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

Example of Association Rules

{Diaper} $\rightarrow$ {Beer},
{Milk, Bread} $\rightarrow$ {Eggs,Coke},
{Beer, Bread} $\rightarrow$ {Milk},

Implication means co-occurrence, not causality!

- What products were often purchased together?— Beer and diapers?!
- What are the subsequent purchases after buying a PC?
- Basket data analysis, cross-marketing, catalog design, sale campaign analysis, Web log (click stream) analysis, and DNA sequence analysis

What Is Frequent Pattern Analysis?

- Frequent pattern: a pattern (a set of items, subsequences, substructures, etc.) that occurs frequently in a data set
- First proposed by Agrawal, Imielinski, and Swami [AIS93] in the context of frequent itemsets and association rule mining
- Motivation: Finding inherent regularities in data
  - What products were often purchased together?— Beer and diapers?!
  - What are the subsequent purchases after buying a PC?
  - What kinds of DNA are sensitive to this new drug?
  - Can we automatically classify web documents?
- Applications
  - Basket data analysis, cross-marketing, catalog design, sale campaign analysis, Web log (click stream) analysis, and DNA sequence analysis.

Why Is Freq. Pattern Mining Important?

- Discloses an intrinsic and important property of data sets
- Forms the foundation for many essential data mining tasks
  - Association, correlation, and causality analysis
  - Sequential, structural (e.g., sub-graph) patterns
  - Pattern analysis in spatiotemporal, multimedia, time-series, and stream data
  - Classification: associative classification
  - Cluster analysis: frequent pattern-based clustering
  - Data warehousing: iceberg cube and cube-gradient
  - Semantic data compression: fascicles
  - Broad applications

Definition: Frequent Itemset

- **Itemset**
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items
- **Support count (σ)**
  - Frequency of occurrence of an itemset
  - E.g. σ({Milk, Bread,Diaper}) = 2
- **Support**
  - **Fraction** of transactions that contain an itemset
  - E.g. s({Milk, Bread, Diaper}) = 2/5
- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a *minsup* threshold

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

Another Format to View the Transaction Data

- Representation of Database
  - horizontal vs vertical data layout

Horizontal
Data Layout

TID	Items
1	A,B,E
2	B,C,D
3	C,E
4	A,C,D
5	A,B,C,D
6	A,E
7	A,B
8	A,B,C
9	A,C,D
10	B

Vertical Data Layout

A	B	C	D	E
1	1	2	2	1
4	2	3	4	3
5	5	4	5	6
6	7	8	9	
7	8	9		
8	10			
9				

Closed Patterns and Max-Patterns

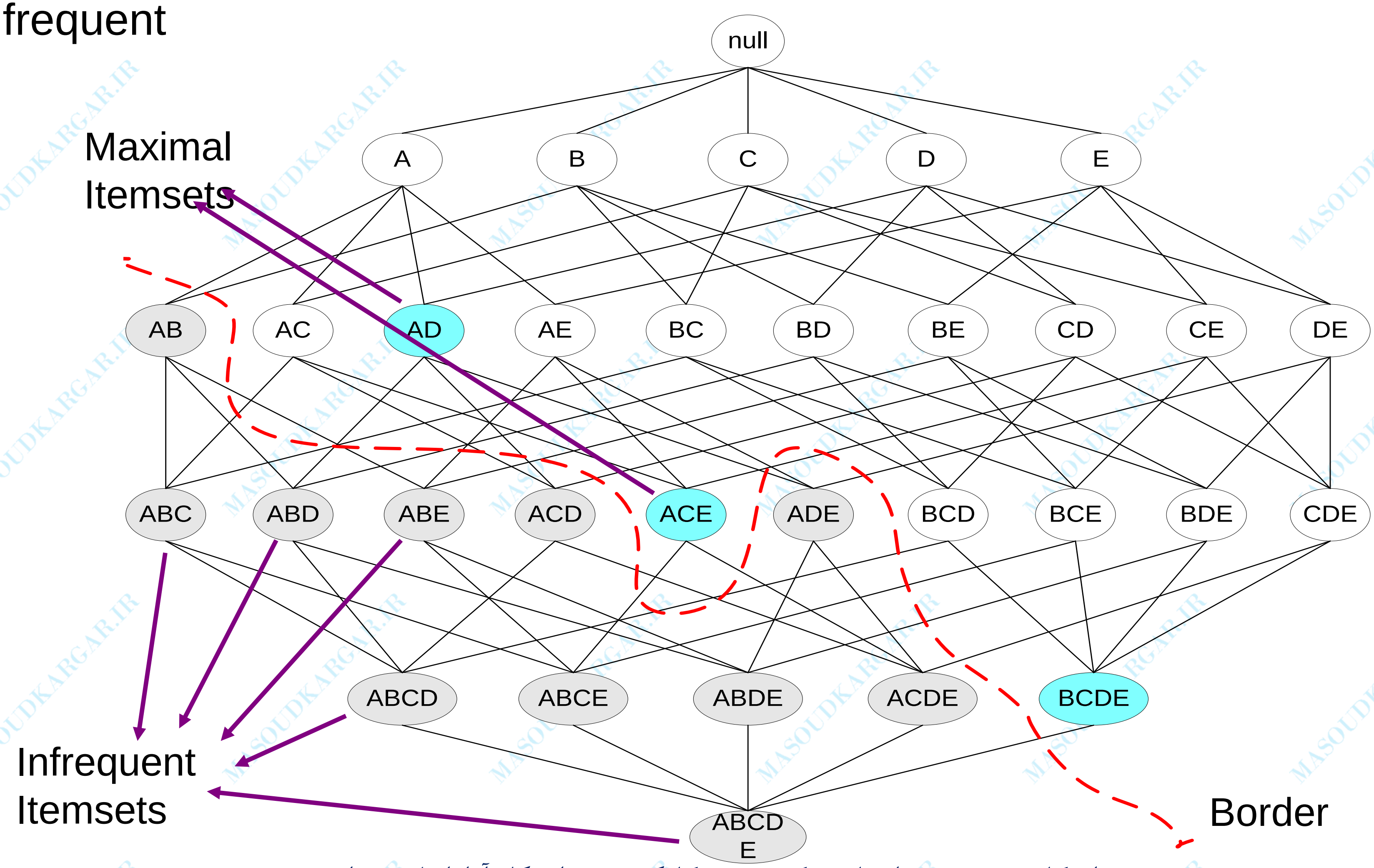
- A long pattern contains a combinatorial number of sub-patterns, e.g., $\{a_1, \ldots, a_{100}\}$ contains $\binom{100}{1} + \binom{100}{2} + \ldots + \binom{100}{100} = 2^{100} - 1 = 1.27 \times 10^{30}$ sub-patterns!

  – (A, B, C)6 frequent → (A, B) 7, (A, C)6, …also frequent

- Solution: *Mine* *closed patterns* *and* *max-patterns* *instead*
  – Closed pattern is a lossless compression of freq. patterns
  – Reducing the # of patterns and rules

Maximal Frequent Itemset

An itemset is maximal frequent if none of its immediate supersets is frequent



Closed Itemset

- An itemset is closed if none of its immediate supersets has the same support as the itemset

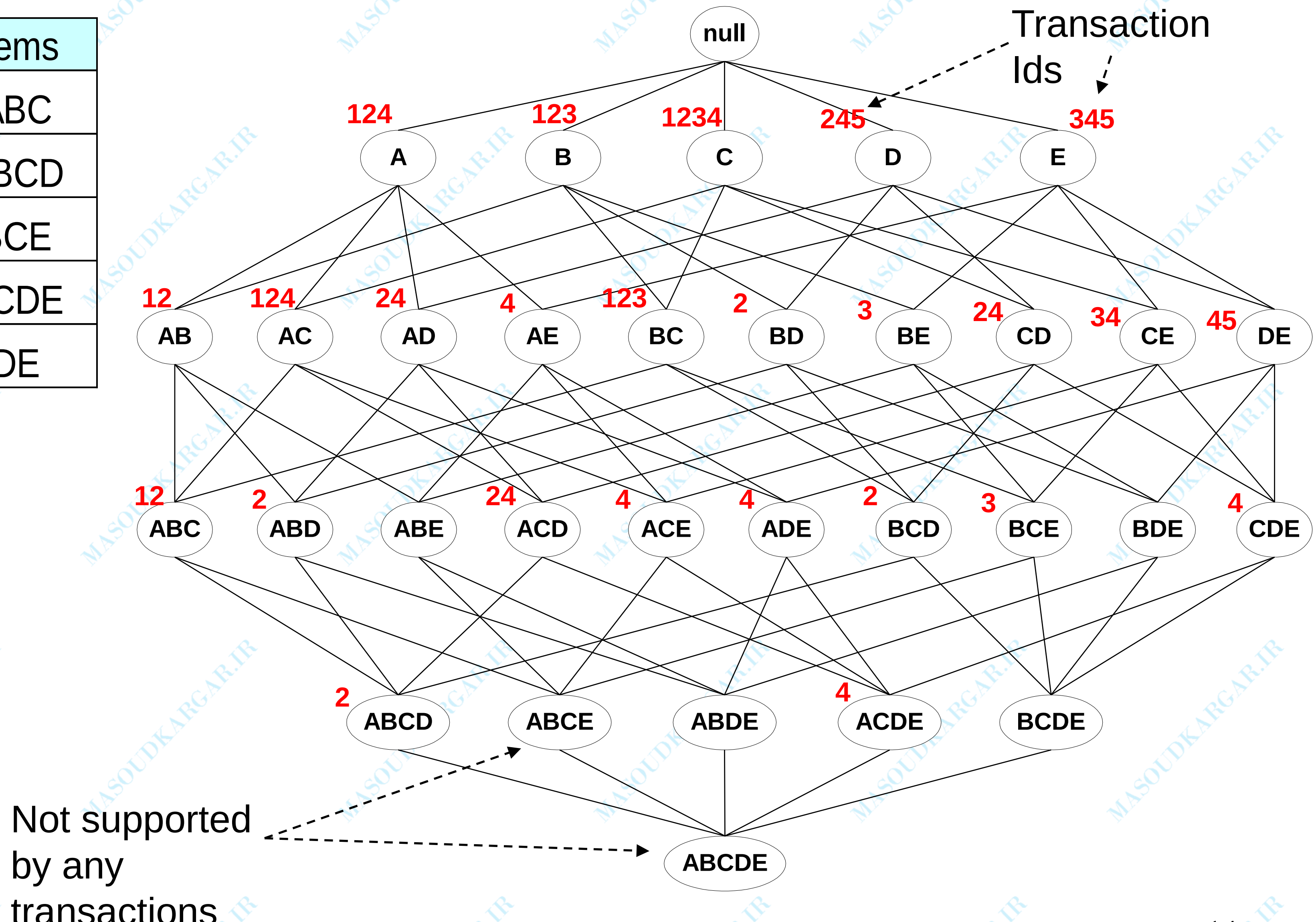
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

Itemset	Support
{A}	4
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	4
{C,D}	3

Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
{B,C,D}	3
{A,B,C,D}	2

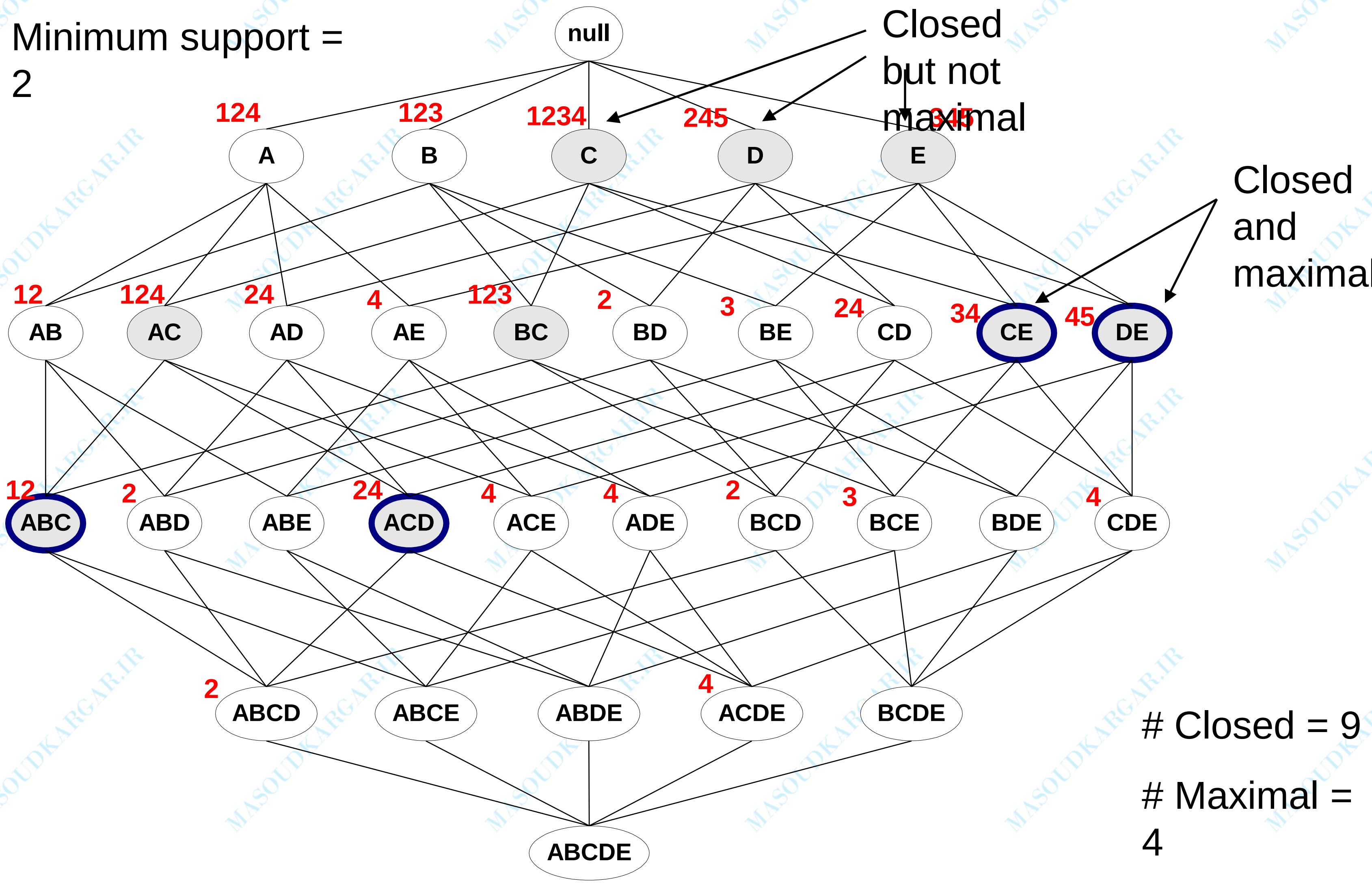
Maximal vs Closed Itemsets

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE



Maximal vs Closed Frequent Itemsets

Minimum support = 2



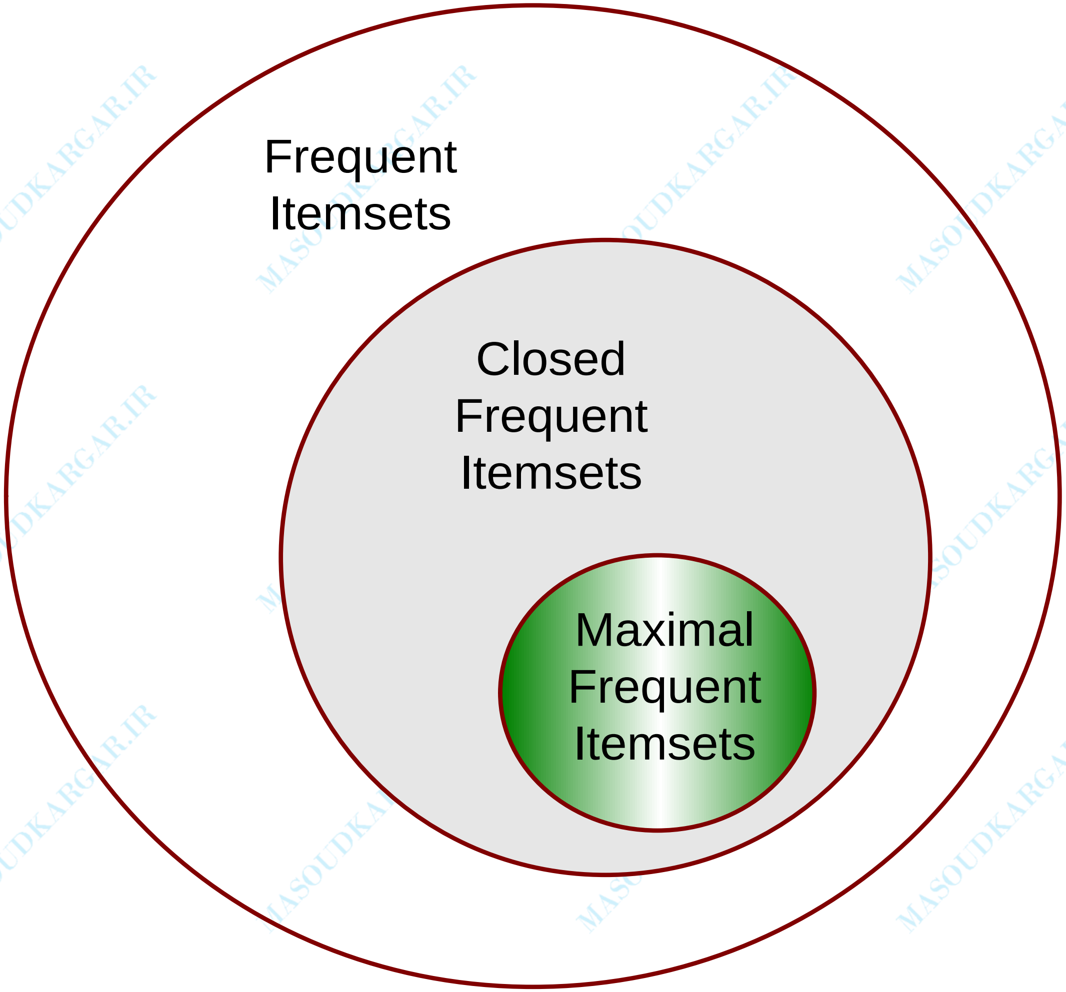
Closed = 9

Maximal = 4

Closed Patterns and Max-Patterns

- Exercise. DB = $\{<a_1, \ldots, a_{100}>, < a_1, \ldots, a_{50}>\}$
  - Min_sup = 1.
- What is the set of closed itemset?
  - $<a_1, \ldots, a_{100}>$: 1
  - $< a_1, \ldots, a_{50}>$: 2
- What is the set of max-pattern?
  - $<a_1, \ldots, a_{100}>$: 1
- What is the set of all patterns?
  - !!

Maximal vs Closed Itemsets



Scalable Methods for Mining Frequent Patterns

- The downward closure property of frequent patterns
  - Any subset of a frequent itemset must be frequent
  - If **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**
  - i.e., every transaction having {beer, diaper, nuts} also contains {beer, diaper}
- Scalable mining methods: Three major approaches
  - Apriori (Agrawal & Srikant@VLDB'94)
  - Freq. pattern growth (FPgrowth—Han, Pei & Yin @SIGMOD'00)
  - Vertical data format approach (Charm—Zaki & Hsiao @SDM'02)

Apriori: A Candidate Generation-and-Test Approach

- Apriori pruning principle: If there is any itemset which is infrequent, its superset should not be generated/tested! (Agrawal & Srikant @VLDB'94, Mannila, et al. @ KDD' 94)
- Method:
  - Initially, scan DB once to get frequent 1-itemset
  - Generate length (k+1) candidate itemsets from length k frequent itemsets
  - Test the candidates against DB
  - Terminate when no frequent or candidate set can be generated

The Apriori Algorithm—An Example

$Sup_{min} = 2$

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1[st] scan →

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

→ L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

← 2[nd] scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

← L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

3[rd] scan →

L_3

Itemset	sup
{B, C, E}	2

The Apriori Algorithm

- Pseudo-code:

C_k: Candidate itemset of size k
L_k : frequent itemset of size k

L_1 = {frequent items};
for (k = 1; L_k !=∅; k++) **do begin**
  C_{k+1} = candidates generated from L_k;
  for each transaction t in database do
    increment the count of all candidates in C_{k+1} that are contained in t
  L_{k+1} = candidates in C_{k+1} with min_support
  end
return $\cup_k L_k$;

Important Details of Apriori

- How to generate candidates?
  - Step 1: self-joining L_k
  - Step 2: pruning
- How to count supports of candidates?
- Example of Candidate-generation
  - L_3={*abc, abd, acd, ace, bcd*}
  - Self-joining: L_3*L_3
    - *abcd* from *abc* and *abd*
    - *acde* from *acd* and *ace*
    - *We cannot join ace and bcd –to get 4-itemset*
  - Pruning:
    - *acde* is removed because *ade* is not in L_3
  - C_4={*abcd*}

How to Generate Candidates?

- Suppose the items in L_{k-1} are listed **in an order**
- Step 1: self-joining L_{k-1}

  insert into C_k

  select $p.item_1, p.item_2, \ldots, p.item_{k-1}, q.item_{k-1}$

  from L_{k-1} p, L_{k-1} q

  where $p.item_1=q.item_1, \ldots, p.item_{k-2}=q.item_{k-2}, p.item_{k-1} < q.item_{k-1}$

- Step 2: pruning

  forall ***itemsets c in C_k*** do

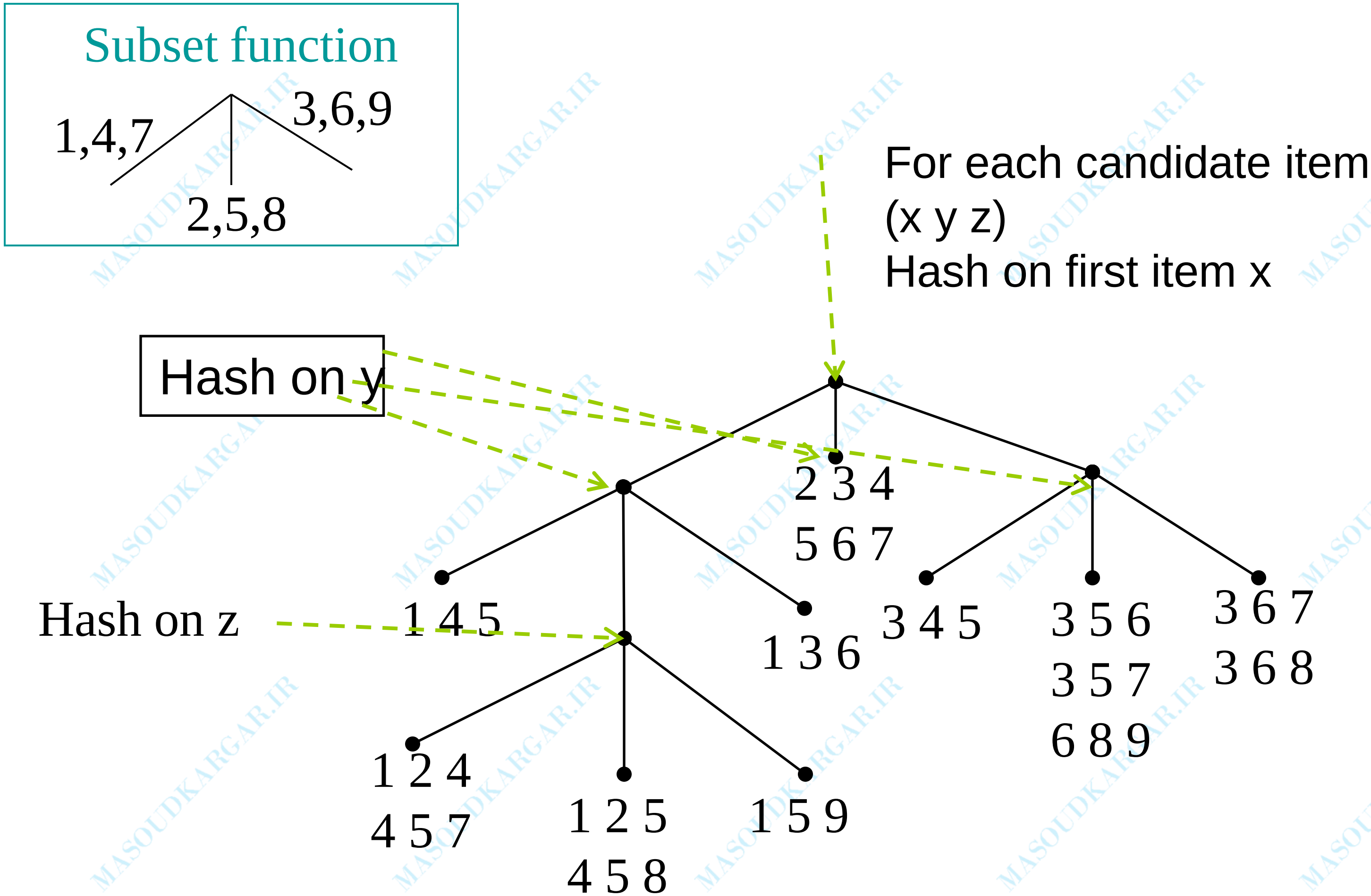
      forall ***(k-1)-subsets s of c*** do

          if *(s is not in L_{k-1})* **then delete** *c* **from** C_k

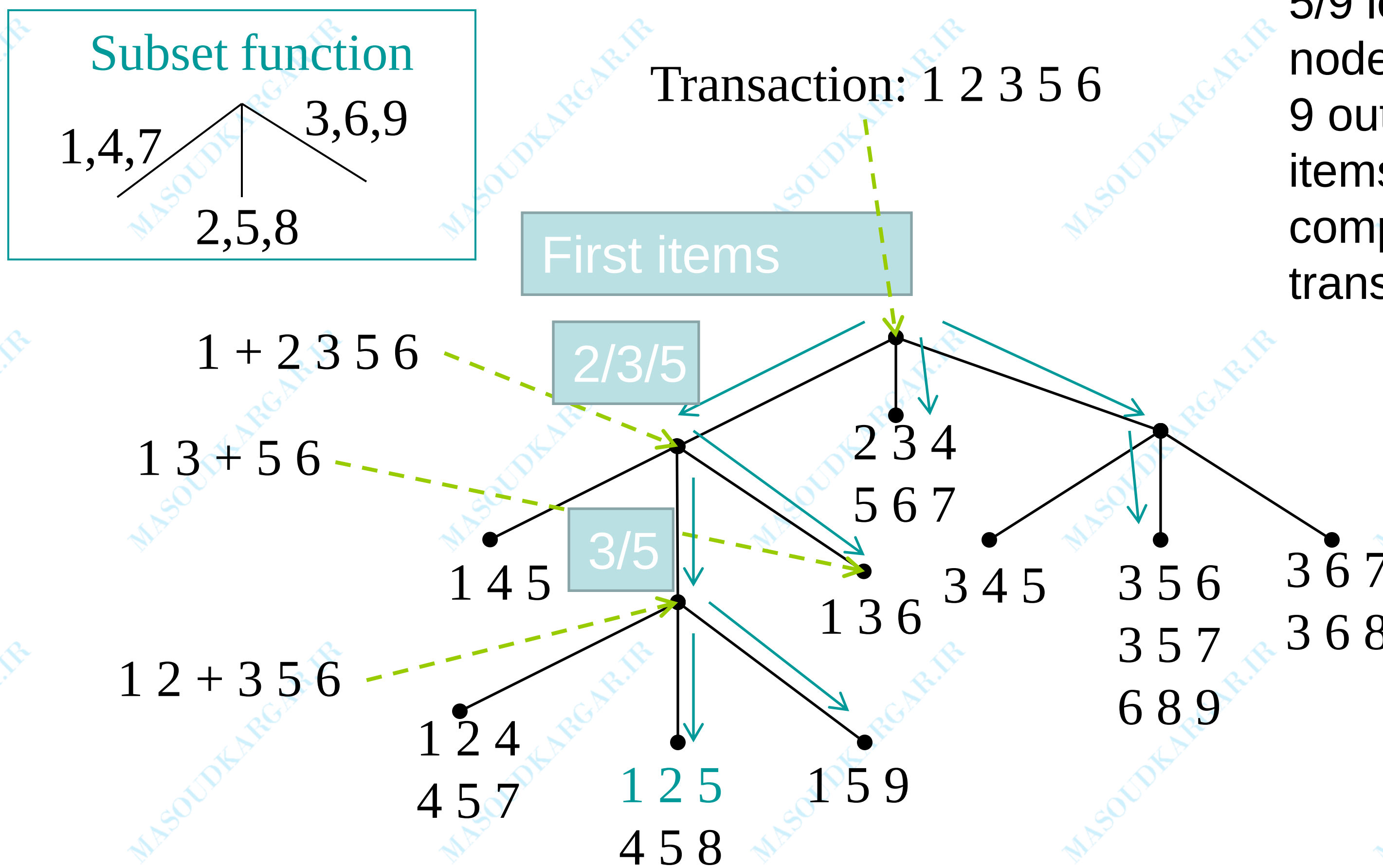
How to Count Supports of Candidates?

- Why counting supports of candidates a problem?
  - The total number of candidates can be very huge
  - One transaction may contain many candidates
- Method:
  - Candidate itemsets are stored in a *hash-tree*
  - *Leaf* node of hash-tree contains a list of itemsets and counts
  - *Interior* node contains a hash table
  - *Subset function*: finds all the candidates contained in a transaction

Example: Store candidate itemsets into Hashtree



Example: Counting Supports of Candidates



5/9 leaf nodes visited
9 out of 15 itemsets compared to transaction

Challenges of Frequent Pattern Mining

- Challenges
  - Multiple scans of transaction database
  - Huge number of candidates
  - Tedious workload of support counting for candidates
- Improving Apriori: general ideas
  - Reduce passes of transaction database scans
  - Shrink number of candidates
  - Facilitate support counting of candidates

Bottleneck of Frequent-pattern Mining

- Multiple database scans are costly
- Mining long patterns needs many passes of scanning and generates lots of candidates
  - To find frequent itemset $i_1 i_2 \ldots i_{100}$
    - # of scans: 100
    - # of Candidates: $\binom{100}{1} + \binom{100}{2} + \ldots + \binom{100}{100} = 2^{100}-1 = 1.27*10^{30}$!
- Bottleneck: candidate-generation-and-test
- Can we avoid candidate generation?

Mining Frequent Patterns Without Candidate Generation

- Grow long patterns from short ones using local frequent items
  - "abc" is a frequent pattern
  - Get all transactions having "abc": DB|abc
  - "d" is a local frequent item in DB|abc → abcd is a frequent pattern

FP-growth Algorithm

- Use a compressed representation of the database using an FP-tree

- Once an FP-tree has been constructed, it uses a recursive divide-and-conquer approach to mine the frequent itemsets

Construct FP-tree from a Transaction Database

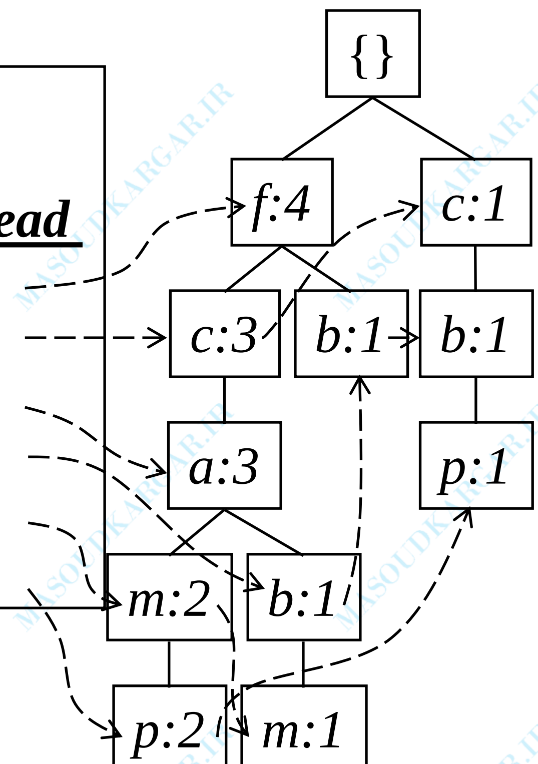
TID	*Items bought*	*(ordered) frequent items*
100	***{f, a, c, d, g, i, m, p}***	***{f, c, a, m, p}***
200	***{a, b, c, f, l, m, o}***	***{f, c, a, b, m}***
300	***{b, f, h, j, o, w}***	***{f, b}***
400	***{b, c, k, s, p}***	***{c, b, p}***
500	***{a, f, c, e, l, p, m, n}***	***{f, c, a, m, p}***

min_support = 3

1. Scan DB once, find frequent 1-itemset (single item pattern)
2. Sort frequent items in frequency descending order, f-list
3. Scan DB again, construct FP-tree

Header Table

Item	*frequency*	*head*
f	*4*	
c	*4*	
a	*3*	
b	*3*	
m	*3*	
p	*3*	



F-list=f-c-a-b-m-p

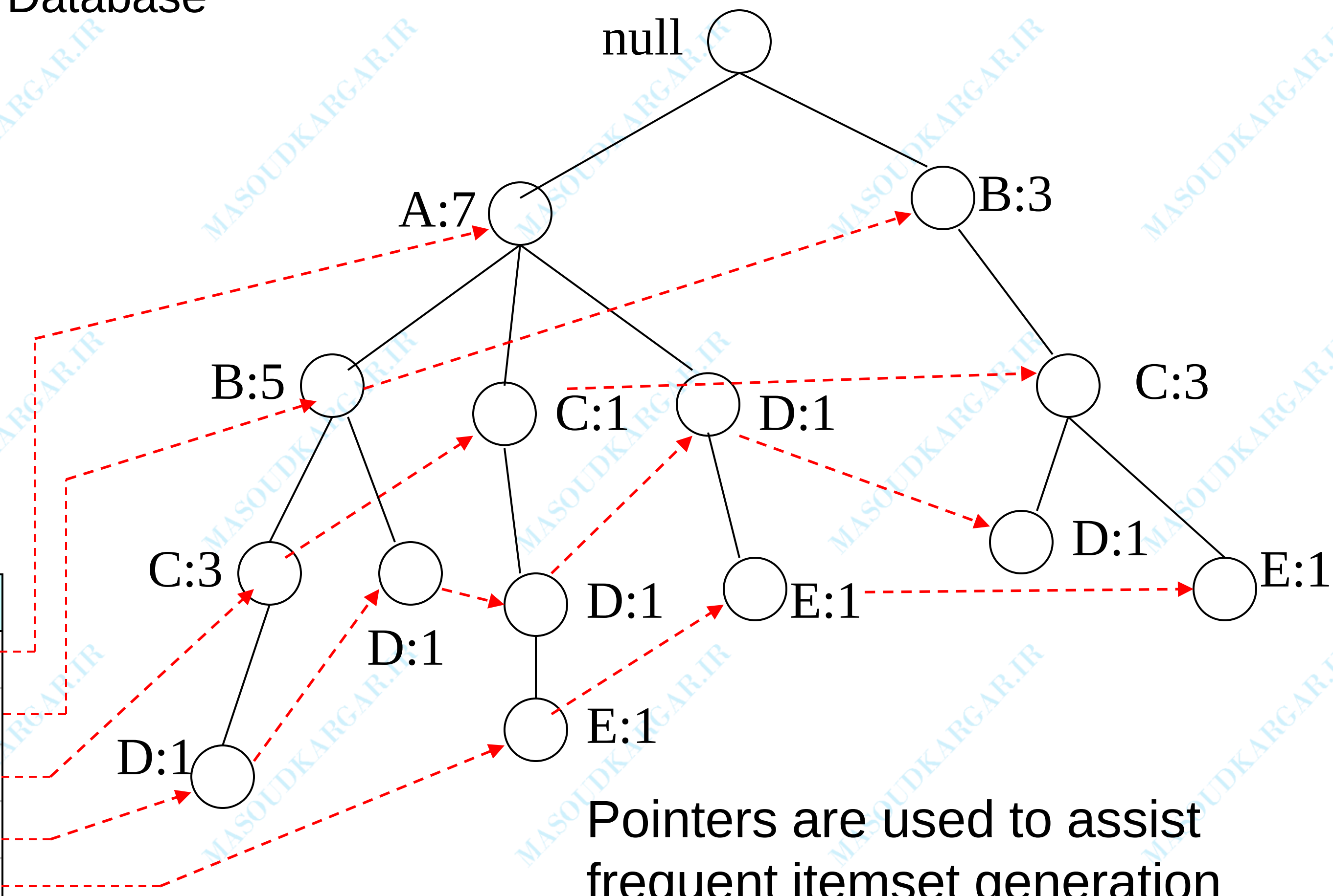
FP-Tree Construction Example

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

Transaction Database

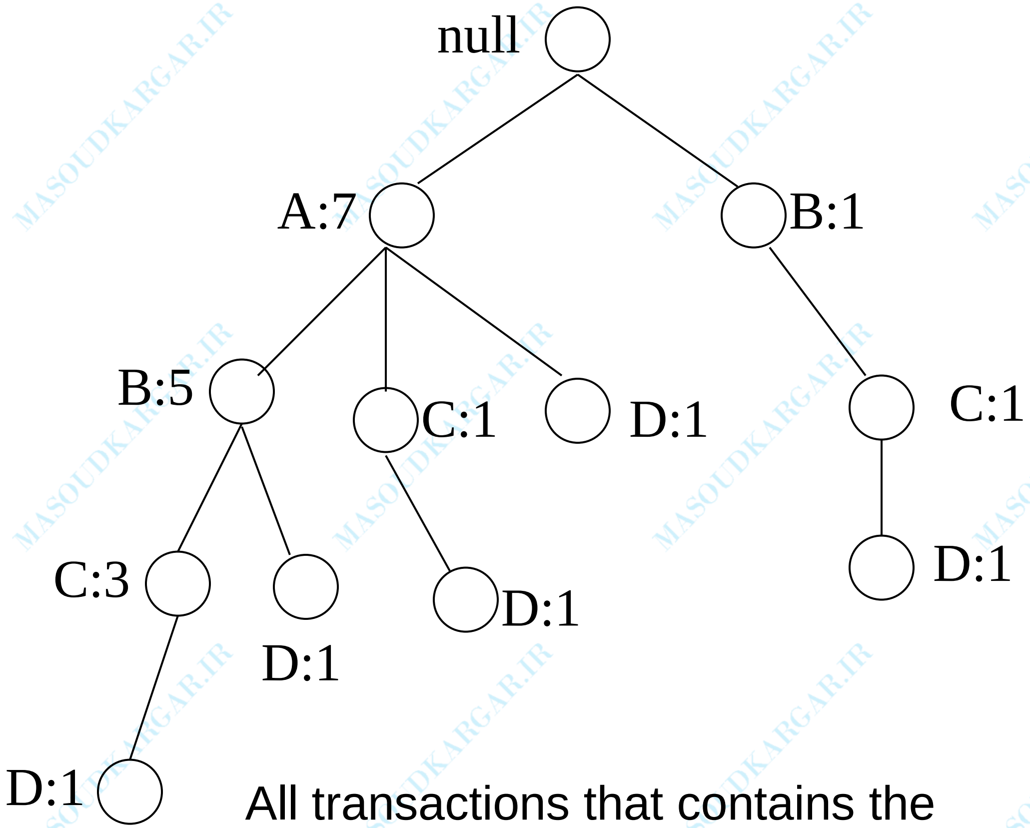
Header table

Item	Pointer
A	
B	
C	
D	
E	



Pointers are used to assist frequent itemset generation

FP-growth



All transactions that contains the patterns ending with D are encapsulated in this tree.

Conditional Pattern base for D:

P = {(A:1,B:1,C:1),
(A:1,B:1),
(A:1,C:1),
(A:1),
(B:1,C:1)}

Recursively apply FP-growth on P

Frequent Itemsets found (with sup > 1):

AD, BD, CD, ACD, BCD

Benefits of the FP-tree Structure

- Completeness
  - Preserve complete information for frequent pattern mining
  - Never break a long pattern of any transaction
- Compactness
  - Reduce irrelevant info—infrequent items are gone
  - Items in frequency descending order: the more frequently occurring, the more likely to be shared
  - Never be larger than the original database (not count node-links and the *count* field)
  - For Connect-4 DB, compression ratio could be over 100

Why Is FP-Growth the Winner?

- Divide-and-conquer:
  - decompose both the mining task and DB according to the frequent patterns obtained so far
  - leads to focused search of smaller databases
- Other factors
  - no candidate generation, no candidate test
  - compressed database: FP-tree structure
  - no repeated scan of entire database
  - basic ops—counting local freq items and building sub FP-tree, no pattern search and matching

Implications of the Methodology

- Mining closed frequent itemsets and max-patterns
  - CLOSET (DMKD'00)
- Mining sequential patterns
  - FreeSpan (KDD'00), PrefixSpan (ICDE'01)
- Constraint-based mining of frequent patterns
  - Convertible constraints (KDD'00, ICDE'01)
- Computing iceberg data cubes with complex measures
  - H-tree and H-cubing algorithm (SIGMOD'01)

MaxMiner: Mining Max-patterns

- 1st scan: find frequent items
  - A, B, C, D, E
- 2nd scan: find support for
  - AB, AC, AD, AE, ABCDE
  - BC, BD, BE, BCDE
  - CD, CE, CDE, DE,

Tid	Items
10	A,B,C,D,E
20	B,C,D,E,
30	A,C,D,F

Potential max-patterns

- Since BCDE is a max-pattern, no need to check BCD, BDE, CDE in later scan
- R. Bayardo. Efficiently mining long patterns from databases. In *SIGMOD'98*

Roadmap

- Frequent Itemset Mining Problem
- Closed itemset, Maximal itemset
- Apriori Algorithm
- FP-Growth: itemset mining without candidate generation
- Association Rule Mining

Definition: Association Rule

- Association Rule
  - An implication expression of the form $X \rightarrow Y$, where X and Y are itemsets
  - Example:
    $\{Milk, Diaper\} \rightarrow \{Beer\}$

TID	Items
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

- Rule Evaluation Metrics
  - Support (s)
    - Fraction of transactions that contain both X and Y
  - Confidence (c)
    - Measures how often items in Y appear in transactions that contain X

Example:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \text{Beer}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|\text{T}|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

Mining Association Rules

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

Example of Rules:

{Milk,Diaper} $\rightarrow$ {Beer} (s=0.4, c=0.67)
{Milk,Beer} $\rightarrow$ {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} $\rightarrow$ {Milk} (s=0.4, c=0.67)
{Beer} $\rightarrow$ {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} $\rightarrow$ {Milk,Beer} (s=0.4, c=0.5)
{Milk} $\rightarrow$ {Diaper,Beer} (s=0.4, c=0.5)

Observations:

- All the above rules are binary partitions of the same itemset:
  {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

Association Rule Mining Task

- Given a set of transactions T, the goal of association rule mining is to find all rules having
  - support ≥ *minsup* threshold
  - confidence ≥ *minconf* threshold

- Brute-force approach:
  - List all possible association rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the *minsup* and *minconf* thresholds

  ⇒ Computationally prohibitive!

Mining Association Rules

- Two-step approach:
  1. Frequent Itemset Generation
     – Generate all itemsets whose support ≥ minsup

  2. Rule Generation
     – Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

- Frequent itemset generation is still computationally expensive

Step 2: Rule Generation

- Given a frequent itemset L, find all non-empty subsets f $\subset$ L such that f $\rightarrow$ L – f satisfies the minimum confidence requirement
  – If {A,B,C,D} is a frequent itemset, candidate rules:

ABC $\rightarrow$D,	ABD $\rightarrow$C,	ACD $\rightarrow$B,	BCD $\rightarrow$A,
A $\rightarrow$BCD,	B $\rightarrow$ACD,	C $\rightarrow$ABD,	D $\rightarrow$ABC
AB $\rightarrow$CD,	AC $\rightarrow$ BD,	AD $\rightarrow$ BC,	BC $\rightarrow$AD,
BD $\rightarrow$AC,	CD $\rightarrow$AB,		

- If |L| = k, then there are $2^k – 2$ candidate association rules (ignoring L $\rightarrow \varnothing$ and $\varnothing \rightarrow$ L)

Rule Generation

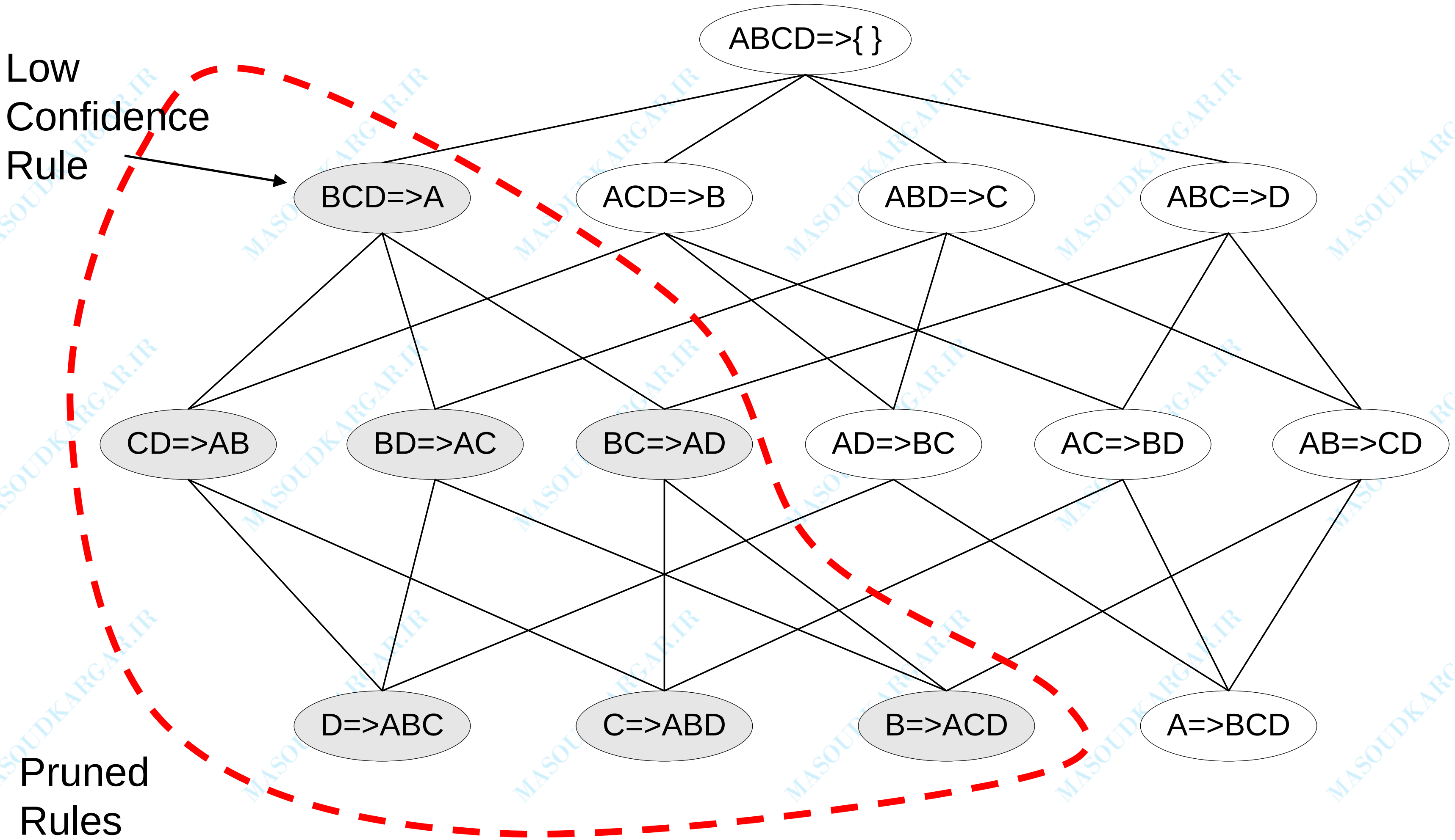
- How to efficiently generate rules from frequent itemsets?
  - In general, confidence does not have an anti-monotone property
    $c(ABC \rightarrow D)$ can be larger or smaller than $c(AB \rightarrow D)$
  - But confidence of rules generated from the same itemset has an anti-monotone property
  - e.g., L = {A,B,C,D}:

$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

    - Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

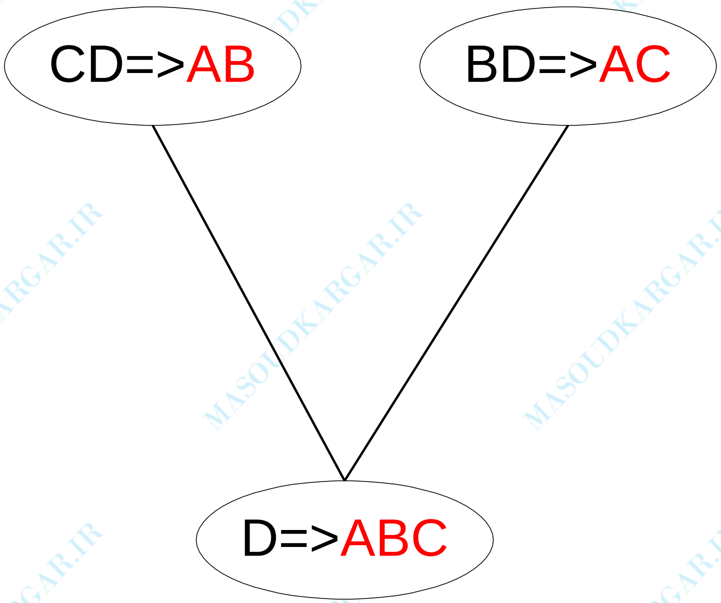
Rule Generation for Apriori Algorithm

Lattice of rules



Rule Generation for Apriori Algorithm

- Candidate rule is generated by merging two rules that share the same prefix
  in the rule consequent
- join(CD=>AB,BD=>AC)
  would produce the candidate
  rule D => ABC
- Prune rule D=>ABC if its
  subset AD=>BC does not have
  high confidence



Pattern Evaluation

- Association rule algorithms tend to produce too many rules
  - many of them are uninteresting or redundant
  - Redundant if $\{A,B,C\} \rightarrow \{D\}$ and $\{A,B\} \rightarrow \{D\}$ have same support & confidence

- Interestingness measures can be used to prune/rank the derived patterns

- In the original formulation of association rules, support & confidence are the only measures used

Computing Interestingness Measure

- Given a rule $X \rightarrow Y$, information needed to compute rule interestingness can be obtained from a contingency table

Contingency table for $X \rightarrow Y$

	Y	$\overline{Y}$	
X	f_{11}	f_{10}	f_{1+}
$\overline{X}$	f_{01}	f_{00}	f_{o+}
	f_{+1}	f_{+0}	$\|T\|$

f_{11}: support of X and Y
f_{10}: support of X and $\overline{Y}$
f_{01}: support of $\overline{X}$ and Y
f_{00}: support of $\overline{X}$ and $\overline{Y}$

Used to define various measures

- support, confidence, lift, Gini, J-measure, etc.

Drawback of Confidence

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea → Coffee

Confidence= P(Coffee|Tea) = 0.75

but P(Coffee) = 0.9

⇒ Although confidence is high, rule is misleading

⇒ $P(\text{Coffee}|\overline{\text{Tea}}) = 0.9375$

Statistical Independence

- Population of 1000 students
  - 600 students know how to swim (S)
  - 700 students know how to bike (B)
  - 420 students know how to swim and bike (S,B)

  - $P(S \wedge B) = 420/1000 = 0.42$
  - $P(S) \times P(B) = 0.6 \times 0.7 = 0.42$

  - $P(S \wedge B) = P(S) \times P(B)$ => Statistical independence
  - $P(S \wedge B) > P(S) \times P(B)$ => Positively correlated
  - $P(S \wedge B) < P(S) \times P(B)$ => Negatively correlated

Statistical-based Measures

- Measures that take into account statistical dependence

$$Lift = \frac{P(Y|X)}{P(Y)}$$

$$Interest = \frac{P(X,Y)}{P(X)P(Y)}$$

$$PS = P(X,Y) - P(X)P(Y)$$

$$\phi - coefficient = \frac{P(X,Y) - P(X)P(Y)}{\sqrt{P(X)[1-P(X)]P(Y)[1-P(Y)]}}$$

Example: Lift/Interest

	Coffee	$\overline{\text{Coffee}}$	
Tea	15	5	20
$\overline{\text{Tea}}$	75	5	80
	90	10	100

Association Rule: Tea $\rightarrow$ Coffee

Confidence= P(Coffee|Tea) = 0.75

but P(Coffee) = 0.9

$\Rightarrow$ Lift = 0.75/0.9= 0.8333 (< 1, therefore is negatively associated)

Drawback of Lift & Interest

	Y	$\bar{Y}$	
X	10	0	10
$\bar{X}$	0	90	90
	10	90	100

	Y	$\bar{Y}$	
X	90	0	90
$\bar{X}$	0	10	10
	90	10	100

$$Lift = \frac{0.1}{(0.1)(0.1)} = 10$$

$$Lift = \frac{0.9}{(0.9)(0.9)} = 1.11$$

Statistical independence:

If P(X,Y)=P(X)P(Y) => Lift = 1

There are lots of measures proposed in the literature

Some measures are good for certain applications, but not for others

What criteria should we use to determine whether a measure is good or bad?

What about Apriori-style support based pruning? How does it affect these measures?

#	Measure	Formula
1	ϕ-coefficient	$\frac{P(A,B)-P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}}$
2	Goodman-Kruskal's (λ)	$\frac{\sum_j \max_k P(A_j,B_k)+\sum_k \max_j P(A_j,B_k)-\max_j P(A_j)-\max_k P(B_k)}{2-\max_j P(A_j)-\max_k P(B_k)}$
3	Odds ratio (α)	$\frac{P(A,B)P(\overline{A},\overline{B})}{P(A,\overline{B})P(\overline{A},B)}$
4	Yule's Q	$\frac{P(A,B)P(\overline{AB})-P(A,\overline{B})P(\overline{A},B)}{P(A,B)P(\overline{AB})+P(A,\overline{B})P(\overline{A},B)} = \frac{\alpha-1}{\alpha+1}$
5	Yule's Y	$\frac{\sqrt{P(A,B)P(\overline{AB})}-\sqrt{P(A,\overline{B})P(\overline{A},B)}}{\sqrt{P(A,B)P(\overline{AB})}+\sqrt{P(A,\overline{B})P(\overline{A},B)}} = \frac{\sqrt{\alpha}-1}{\sqrt{\alpha}+1}$
6	Kappa (κ)	$\frac{P(A,B)+P(\overline{A},\overline{B})-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A)P(B)-P(\overline{A})P(\overline{B})}$
7	Mutual Information (M)	$\frac{\sum_i \sum_j P(A_i,B_j)\log\frac{P(A_i,B_j)}{P(A_i)P(B_j)}}{\min(-\sum_i P(A_i)\log P(A_i), -\sum_j P(B_j)\log P(B_j))}$
8	J-Measure (J)	$\max\Big(P(A,B)\log(\frac{P(B\|A)}{P(B)}) + P(A\overline{B})\log(\frac{P(\overline{B}\|A)}{P(\overline{B})}),$ $P(A,B)\log(\frac{P(A\|B)}{P(A)}) + P(\overline{A}B)\log(\frac{P(\overline{A}\|B)}{P(\overline{A})})\Big)$
9	Gini index (G)	$\max\Big(P(A)[P(B\|A)^2 + P(\overline{B}\|A)^2] + P(\overline{A})[P(B\|\overline{A})^2 + P(\overline{B}\|\overline{A})^2]$ $-P(B)^2 - P(\overline{B})^2,$ $P(B)[P(A\|B)^2 + P(\overline{A}\|B)^2] + P(\overline{B})[P(A\|\overline{B})^2 + P(\overline{A}\|\overline{B})^2]$ $-P(A)^2 - P(\overline{A})^2\Big)$
10	Support (s)	$P(A,B)$
11	Confidence (c)	$\max(P(B\|A), P(A\|B))$
12	Laplace (L)	$\max\Big(\frac{NP(A,B)+1}{NP(A)+2}, \frac{NP(A,B)+1}{NP(B)+2}\Big)$
13	Conviction (V)	$\max\Big(\frac{P(A)P(\overline{B})}{P(A\overline{B})}, \frac{P(B)P(\overline{A})}{P(B\overline{A})}\Big)$
14	Interest (I)	$\frac{P(A,B)}{P(A)P(B)}$
15	cosine (IS)	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
16	Piatetsky-Shapiro's (PS)	$P(A,B) - P(A)P(B)$
17	Certainty factor (F)	$\max\Big(\frac{P(B\|A)-P(B)}{1-P(B)}, \frac{P(A\|B)-P(A)}{1-P(A)}\Big)$
18	Added Value (AV)	$\max(P(B\|A) - P(B), P(A\|B) - P(A))$
19	Collective strength (S)	$\frac{P(A,B)+P(\overline{AB})}{P(A)P(B)+P(\overline{A})P(\overline{B})} \times \frac{1-P(A)P(B)-P(\overline{A})P(\overline{B})}{1-P(A,B)-P(\overline{AB})}$
20	Jaccard (ζ)	$\frac{P(A,B)}{P(A)+P(B)-P(A,B)}$
21	Klosgen (K)	$\sqrt{P(A,B)}\max(P(B\|A) - P(B), P(A\|B) - P(A))$

Subjective Interestingness Measure

- Objective measure:
  - Rank patterns based on statistics computed from data
  - e.g., 21 measures of association (support, confidence, Laplace, Gini, mutual information, Jaccard, etc).

- Subjective measure:
  - Rank patterns according to user's interpretation
    - A pattern is subjectively interesting if it contradicts the expectation of a user (Silberschatz & Tuzhilin)
    - A pattern is subjectively interesting if it is actionable (Silberschatz & Tuzhilin)

Summary

- Frequent item set mining applications
- Apriori algorithm
- FP-growth algorithm
- Association mining
- Association Rule evaluation

قدردانی

- Dr. Jianjun Hu
  http://mleg.cse.sc.edu/edu/csce822/
- University of South Carolina
- Department of Computer Science and Engineering